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Ultrasonic diffraction by a circular transducer:
Isogeometric analysis sensitivity to full Gauss quadrature points

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Abstract: A successful evaluation of a radiated acoustic field was obtained by discretizing a circular transducer surface into only 20 Bézier elements. Accuracy of results was highly dependent on the number of Gauss points (ngp) used. This number can lead to changing the amplitude of both plane and edge waves, especially for near field points. But most changes have been detected between those waves where unexpected waves appeared. They can be of the same amplitude as the plane wave if ngp is chosen arbitrarily. To improve accuracy, the ngp must be increased gradually from the transducer centre and not only at its edge. © 2020 Acoustical Society of America

1. Introduction
The impulse response method (IRM) (Stepanishen, 1971; Weight, 1984), finite difference method (Alford and Kelly, 1974; Alia et al., 2004), and boundary element method (Alia, 2006), are some techniques that have been successfully used for the simulation of transient ultrasonic diffraction. However, the widely used method of IsoGeometric Analysis (IGA) has not yet been applied to this type of problem, although IGA has proven its effectiveness for time harmonic acoustic problems (Coox et al., 2016; Khajah et al., 2019). This is a method that may seem promising given the success it has had so far, and it deserves to be tested for ultrasonic diffraction.

IGA is based on the use of the same basis functions to describe the domain geometry and to approximate the unknowns of a boundary value problem. Therefore, it creates a link between two initially separated environments—analysis and geometry design. In Aided Computer Design, the use of Bsplines is common to represent geometries. The reason for such common use is mainly due to its flexibility and continuity advantages. However, Bspline spans a parametric space which consists of many elements. Unlike classical element-based techniques such as finite element method and boundary element method, the parametric space of Bspline is local to patches rather than elements. Consequently, connectivity of two adjacent elements overlaps by sharing many internal control points. This connectivity structure complicates linking the geometry to classical element-based techniques. On the other hand, engineers have local control on Bézier-based geometry because changing one control point affects at most two adjacent elements. Borden et al. (2011) proposed the Bézier decomposition technique that describes elements as in element-based techniques, but with a higher continuity by decomposing Bspline basis functions into Bézier ones which are called Bernstein polynomials.

This paper aims to directly solve Rayleigh integral in the time domain by using Bézier decomposition, which convert the original Bspline-based geometry into Bézier elements. Because the geometry is exact, it allows using less elements than the isoparametric approach. However, to be able to successfully evaluate the ultrasonic diffraction, it is necessary to choose the number of Gauss points (ngp) carefully. This current study explores, for the first time, the effect of Gauss points on the results of ultrasonic diffraction. Unlike the other problems in mechanics, Gauss points not only play their role in numerical integration but they also play the role of source points due to the presence of retarded time in Rayleigh integral. An insufficient ngp can lead to a change in the acoustic response of the studied system. A small ngp can generate additional waves of the same amplitude as the diffracted field itself.

2. Method
The Rayleigh integral equation is a specific case of the Helmholtz Kirchhoff integral in which the normal derivative of the Green function is equal to zero. Rayleigh integral represents a
well-established evaluation of the transient acoustic radiation from baffled planar transducers. It can be formulated as a boundary value problem in terms of velocity potential,
\[
\phi(\xi, t) = \int_{S_T} \frac{1}{2\pi r(\xi, x)} V(x, t) dS_T(x),
\]
where \(\xi\) is an observation point, \(x\) is the source point on the surface transducer \(S_T\), \(\rho\) is the fluid density, \(t_r = t - r(\xi, x)/c\) is the retarded time, and \(c\) is the sound velocity in that fluid.

Instead of using observed-centred coordinates system \(R\) and \(\theta\) as done in Harris (1981) and Qian and Harris (2012), the exact geometry is created using Bspline.

Describing geometry requires a set of \(n\) Bspline control points and their corresponding basis functions \(N(u)\) which are constructed on a knot vector \(\{u_1, u_2, \ldots, u_{n+p+1}\}\) for a given order \(p\). The Cartesian coordinates \(T(u)\) of a given location in the parametric coordinate \(u\) is expressed as
\[
T(u) = \sum_{i=1}^{n} N_i(u) P_{Bi},
\]
where \(P_{Bi}\) represents the \(i\)th B spline control point. When Bspline is also used as a discretization technique of analysis, it is even better to develop it from an element-based methods point-of-view. This can be achieved by using the Bézier extraction procedure (Borden et al., 2011). To map the Bernstein polynomial basis on the Bézier element to the global B spline basis, a linear operator \(C_e\) called Bézier extraction operator is to be constructed for each element. It generates a finite set of Bézier element such as
\[
N = C_e B,
\]
where \(N\) is a vector of B spline basis functions and \(B\) is a vector of Bézier basis functions. In the same manner, the transpose of extraction operator maps the B spline control points \(P_{Bi}\) to the Bézier control points \(P_{Bi}\) such that
\[
P_{Bi} = C^T_e P_{Bi}.
\]
The geometry which is initially described by Bspline \((x = P_{Bi}N)\) can be now given by Bézier description \((x = P_{Bi}B)\). Likewise, the acoustic velocity is discretized using basis functions in physical space
\[
V(x, t) = V_{Bi}B = V_{Bi}N,
\]
where \(V_{Bi}\) and \(V_{Bi}\) are control variables in the Bspline and Bézier description, respectively.

Here, since the profile is uniform, that is all transducer elements are excited by the same velocity, and according to the partition unity of basis functions then \(V_{Bi}\) and \(V_{Bi}\) also represent the physical acoustic velocity which is not the case if the profile is not uniform.

Now, since the geometry is composed from Bézier elements, the Rayleigh integral can be solved numerically by a summation of contribution of each element but with its corresponding retarded time. The integration is performed with full Gauss quadrature rule.

3. Results and discussion

To perform simulations, a circular transducer of a radius \(a\) and a uniform vibration profile were considered. It was excited by an impulsive velocity \(V(t)\) of duration \(\tau\) and a maximum \(V_{\text{max}}\). The radiated ultrasonic field into a homogeneous and lossless fluid was calculated for some observation points located at different positions \(z\) from the transducer centre. Time, position, and pressure were normalized with respect to \(alc\) a, and \(P_{\text{ref}} = \rho c V_{\text{max}}\), respectively. Finally, in what follows ngp is the ngp per Bézier element and \(D\) is the surface density of ngp which is calculated as the ratio between (1) ngp per element and (2) the element surface. \(D\) is expressed in (ngp/ mm\(^2\)). Both on- and off-axis fields are considered in this study.

3.1 On-axis field

As shown in Fig. 1(a), a good agreement between IRM and IGA was obtained for a point field situated at \(z/a = 2.5\) on-axis. The radiated field was constituted by two waves of the same amplitude. Their normalized arrival times are given by \(t_1 = z/a\) and \(t_2 = \sqrt{z^2 + a^2}/a\), respectively. The first wave, called plane wave, has the same waveform as the excitation velocity. The second wave, called edge wave, is an inverse replica of the plane wave. Between these two waves, no acoustic pressure was detected. This result was obtained with only 20 cubic Bézier elements [as shown in Fig. 1(b)] by adopting a large density of ngp \((D = 42.4)\).

The first set of simulations aimed to examine the effect of the ngp on the error of the amplitudes of both plane and edge waves, which are denoted by \(\varepsilon_1\) and \(\varepsilon_2\), respectively. For this
purpose, several simulations have been conducted. On average, \( \varepsilon_1 \) was less than \( \varepsilon_2 \). \( \varepsilon_1 \) fluctuated slightly around a value when \( n_{gp} \) increased, then it stabilized at that value for large \( n_{gp} \). When \( n_{gp} \) was increasing, \( \varepsilon_2 \) showed a fluctuating variation around an average value. This average value \((\varepsilon_2/C_0 = 0.6\%)\) was greater than the value at which \( \varepsilon_1 \) has stabilized \((\varepsilon_1/C_0 = 3\%)\).

The second set of simulations aimed to examine the effect of \( n_{gp} \) on the error in the wave amplitude propagating between \( t_1 + \tau \) and \( t_2 \), i.e., wave between the plane and edge wave because no pressure has to be present at that range in case of a uniform vibration profile. What stood out from simulations was the presence of additional non-physical waves between the plane and edge waves.

Figure 1(c) shows the variation, as a function of \( z/a \), of the total \( n_{gp} \) normalized with respect to the total \( n_{cp} \) for two maintained values of \( \varepsilon_0 \), 0.25% and 0.45% and for both quadratic and cubic Bézier elements. These errors are calculated with respect to the amplitude of the plane wave \( P_{\text{plane}} \) by using the following formula: \[ \frac{1}{N} \sum_{i=1}^{N} P_i / P_{\text{plane}} \] where \( N \) is the number of samplings in \([t_1 + \tau, t_2]\) and \( P_i \) is the calculated non-zero pressure at that range. As expected, it can be seen from the variations of Fig. 1(c) that (1) an error of 0.25% required more \( n_{gp} \) that what required the error of 0.45% and (2) for a given error \( \varepsilon_0 \), the quadratic Bézier element required more than what required cubic Bézier element.

The most interesting aspect of Fig. 1(c) is the fact that \( n_{gp_{\text{Total}}}/n_{cp_{\text{Total}}} \) decreased with increasing \( z/a \) to maintain the same error \( \varepsilon_0 \). It is known that IGA offers a better accuracy because the geometry is exact. Here, it is not only a matter of integration over an exact geometry, but also of interference of waves arriving at different times. The lack of a sufficient number of waves to interfere, which is related to \( n_{gp} \) in this case, leads to a modification of the amplitude of the waves. This is less pronounced for plane wave because it comes directly from the centre of transducer and its nearest neighborhood. But, the edge wave and the pressure between plane and edge waves are the results of the waves radiated by the entire transducer and are therefore directly influenced by \( n_{gp} \) to maintain that error.

For near field points, \([t_1 + \tau, t_2]\) range is large and the distance between source points on the transducer and observation points on-axis are both small and too different from one source point to another. In this case, the insufficiency in the number of source points \((n_{gp})\) leads to apparition of additional waves between plane and edge waves. However, far from transducer \([z/a \in [2, 3]]\) in Fig. 1(c)] the range \([t_1 + \tau, t_2]\) is smaller and the distance between source points
and observation points are almost the same. So even with a small ngp, one can achieve a good precision at points far from the transducer. As shown in Fig. 1(d), what matters most is the surface density of ngp. To maintain the same error ($\varepsilon_0 \approx 0.25\%$) for different meshes (20, 45, and 80 elements) when $z/a$ increased, the decrease of Gauss points density was the same for all meshes.

The third set of simulations aimed to investigate the effect of a small ngp on the radiated field and to understand the origin of the non-physical waves. To this end, 20 cubic Bézier elements and a Gauss points density of $14 \text{mm}^{-2}$ were chosen ($a = 12 \text{mm}$). Interestingly, an unexpected waveform of those waves was obtained. Plane and edge waves were separated by three perfectly distinct waves as shown in Fig. 2(a) for $z/a = 0.5$. The amplitude of the first wave was too small and equal to 6.25% of the plane wave amplitude. While the maximum of the second wave was equal to half of the plane wave amplitude, the maximum of the third wave was almost equal to the plane wave amplitude.

Consequently, a small ngp can lead to a completely wrong result. Even the mesh was coarser, the choice of 20 Bézier elements [Fig. 1(b)] was motivated by the ease of transducer decomposition into three zones: a central zone (1) represented by a square in Fig. 1(b), a peripheral zone (3) including all elements with a perfectly circular edge, and an intermediate zone (2) delimited by zones (1) and (3). This decomposition allowed to assign different ngp for each zone and therefore to investigate their influence on the acoustic pressure.

In order to understand how these non-physical waves occurred and how to minimize them, different ngp and therefore Gauss point densities have been used to perform simulations.

As can be seen in Figs. 2(b)–2(d), increasing Gauss points density in zones (1), (2), and (3) allowed the non-physical waves (1), (2), and (3) to disappear, respectively. This proves that the origin of wave (i) was zone (i). It is true that the greatest error comes from the transducer edge but the error whose origin is zones (1) and (2) is not negligible either. These same features have also been observed for different field points apart that the maximum of the three non-physical waves decreased when $z/a$ increased. Consequently, to eliminate all non-physical waves, it is necessary to gradually increase ngp from the centre to the edge of the transducer. As the number of elements increases, the ngp decreases [Fig. 1(d)]. Consequently, the gradual refinement of the mesh can also be carried out instead of the gradual increase in the ngp.

According to this last set of simulations, $\varepsilon_2$ was only affected by varying ngp in zone (3) and $\varepsilon_1$ was only influenced by varying it in zone (1). The change of ngp in only zone (2) did not

Fig. 2. Transient ultrasonic field radiated—at an observation point on-axis $z/a = 0.5$—by a circular transducer constituted by 20 cubic Bézier elements and divided into 3 zones: zone (1) is the central square in Fig. 1(b), zone (3) is constituted by all elements at the transducer edge, zone (2) located between zones (1) and (3). (a) The same density for all zones, (b) higher density in zone (1), (c) higher density in zone (2), and (d) higher density in zone (3). The surface density $D_i$ in zone (i) is expressed in ngp/mm$^2$ for a radius of $a = 12 \text{ mm}$.
change the value of these two errors. Of course, increasing ngp in any zone (i) resulted in a decrease of the maximum of the non-physical wave (i).

3.2 Off-axis field

The on-axis field simulations were performed by using a circular transducer. The high symmetry of the on-axis field case allows identifying the origin of the non-physical waves between $t_1$ and $t_2$ in three zones. To examine if the general conclusions given for the on-axis field still apply for the off-axis one, three other sets of simulations were performed.

The first set of simulations was performed for a point field located at $z/a = r/a = 0.5$. Figure 3(a) represents the radiated field at that point when a single low density was used for all regions ($D = 14$). The effect of using a low density in each zone was shown in Figs. 3(b), 3(c), and 3(d). According to Figs. 3(b)–3(d), all zones contributed to the generation of non-physical waves between the plane and the first edge wave. However, only zone (3) contributes to their generation between the two edge waves [Fig. 3(d)]. In the absence of symmetry, the influence of each zone can no longer be located exactly as it was for the on-axis field. The effect of each zone was extended rather than limited in time.

The second set of simulations have been performed, by using $D_1 = 76$, $D_2 = 77$, and $D_3 = 14$ as densities, and by increasing $r/a$ gradually from 0 to 1.2, to understand how extension in time happened in case of the third non-physical wave for $z/a = 0.5$.

In the vicinity of the axis of symmetry ($r/a = 0.005$), the field (including the non-physical waves) was similar to the one on the axis except that the amplitude of the edge wave and the third non-physical wave decreased. Near axis, the edge wave split into 2 edge waves of smaller amplitude. At $r/a = 0.03$, the third non-physical wave started to interfere with the first edge wave and the maximum reached by this non-physical wave was of the same order of the edge wave. At $r/a = 0.05$, the third non-physical wave interfered also with the second edge wave and kept a maximum in the order of the edge wave amplitude.

As $r$ increased, the time separating the edge waves and the extension over time of the non-physical wave increased. As long as the plane and the first edge wave were separated ($r/a \approx 0.7$), the maximum of the third non-physical wave was in the order of the edge wave amplitude. However, once the plane and the first edge wave started to interfere (at $r/a = 0.8$), the third non-physical wave saw its maximum increasing until it reached the amplitude of the plane wave (at $r/a = 0.95$). Of course, obtaining a good result without non-physical wave using a

![Fig. 3. Transient ultrasonic field radiated—at an observation point off-axis $z/a = r/a = 0.5$—by a circular transducer constituted by 20 cubic Bézier elements and divided into 3 zones: zone (1) is the central square in Fig. 1(b), zone (3) is constituted by all elements at the transducer edge, and zone (2) located between zones (1) and (3). (a) The same density for all zones, (b) lower density in zone (1), (c) lower density in zone (2), and (d) lower density in zone (3). The surface density $D_i$ in zone (i) is expressed in ngp/mm² for a radius of $a = 12$ mm.](https://doi.org/10.1121/10.0000591)
large density of $D_3 = 78$ (instead of 14) was a good guarantee that zone (3) was the only origin of the non-physical wave.

Off-axis, but in shadow region (at $r/a \geq 1$), the plane wave disappeared. The field is only composed from two edge waves that the amplitude is smaller than the plane wave. According to simulations, the maximum of the non-physical wave was up to 6 times bigger than the edge wave. Here, however, using $D_3 = 78$ did not produce a good result without non-physical waves. Increasing the density in all zones allowed minimizing the maximum of these non-physical waves. This proved that (1) the effect of zones (1) and (2) contributed to generating non-physical waves between the edge waves [contrary to the observations made in Figs. 3(b) and 3(c)], and (2) high densities have to be used as $r$ increased particularly in shadow region where the right amplitude of edge waves are difficult to capture.

All simulations were also performed for $z/a = 1$ using the same densities ($D_1 = 76$, $D_2 = 77$, and $D_3 = 14$). The same acoustic response was obtained when $r/a$ has been varied apart that the amplitude of the third non-physical wave was smaller than the case of $z/a = 0.5$ and a significant contribution of zones (1) and (2) appeared further into the shadow region ($r > 1.2$). Moreover, in comparison to near field point ($z/a = 0.5$), less densities are needed to suppress the non-physical waves in case of $z/a = 1$.

Consequently, unlike the on-axis case, it is difficult to separate the effect of the three zones in the off-axis case and to stipulate that such a non-physical wave originates from such a zone. Moreover, it has been found that as $r$ increases, the density must also increase. Nevertheless, for a good representation of the field, (1) decreasing the density when $z$ increases (for a given $r$) and (2) increasing $\text{ngp}$ gradually from the centre to the edge remain two recommendations valid for the off-axis case.

Finally, from a computational point of view, a gradual increase in Gauss points can be very costly in time. Thus, the choice of Gauss quadrature rule may be far from optimal. In mechanics, a lot of effort has been devoted to the development of new optimal (Auricchio et al., 2012) or even reduced (Schillinger et al., 2014) quadrature rules. According to the current results of this study, it appears that a reduced number of integration points should lead to significant errors because these points also play the role of source points. It would be interesting to examine the effect of these emerging quadrature rules within the IGA approach on the ultrasonic diffraction.

4. Conclusion

Simulations were performed to verify whether IGA is a suitable method to predict ultrasonic field radiated by a transducer. The obtained results were in good agreement with IRM when only 20 Bézier elements were used. However, their precision was directly dependent on the $\text{ngp}$ and the position of the field points. It has been found that near field points are more exposed to large errors than far field points. Besides the error in the amplitude of the plane wave and edge wave, additional non-physical waves can appear. Their amplitude can be comparable to that of the plane wave if no precaution, in terms of the $\text{ngp}$, is taken into account beforehand.

References and links


